HALL'S NAUTICAL SLIDE-RULE.

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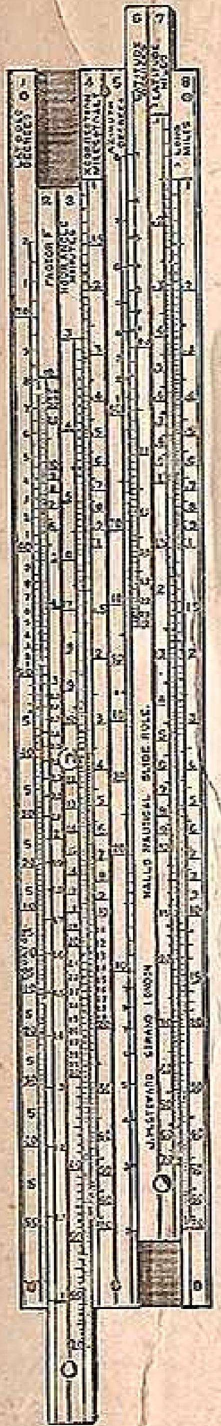


Hall's Nautical Slide-Rule.

The Illustration shows the Instrument set ready for finding Noon position.

Opposite any Hour Angle on the slide, which has been set for Latitude and Declination at Noon, is seen the Ex-Meridian correction to be added to Altitude, to reduce it to Meridian Altitude.

For example :—

At 10m. on Scale 3, is 3′ correction on Scale 4.

At 17½m. on Scale 3, is 9·2′ correction on Scale 4, and without further setting any number of Ex-Meridians may be corrected.

When Latitude at Ex-Meridian is known, the error in Latitude can be found, and the Chronometer sight corrected for this error.

The Instrument is set for Latitude 41° (Scale 6) and Azimuth 63° (Scale 7). Opposite any δ Latitude (Scale 7) is seen the corresponding δ Longitude.

For example :—

Opposite δ Latitude 6′ is 4′ δ Longitude.

Opposite δ Latitude 10′ is 6·7′ δ Longitude.

The Slide-Rule thus saves the Navigator all troublesome arithmetic at the time of day when he is busiest and can least spare time for unnecessary labour.

HALL'S

NAUTICAL SLIDE-RULE.

INVENTED AND DESIGNED

BY

The Rev. WILLIAM HALL, R.N.

INDEX.

In the illustration the Scales are figured 1, 2, 3, 4, 5, 6, 7, 8, and are referred to in the text as, I., II., III., IV., V., VI., VII., VIII.

The following abbreviations are used in the text :—

LAT. . . .	Latitude.
LNG. . . .	Longitude.
DEC. . . .	Declination.
ALT. . . .	Altitude.
EQN. . . .	Equation.
AZ.	Azimuth.
Varn. . . .	Variation.
REF. . . .	Refraction.
DEP. . . .	Departure.
Chron. . .	Chronometer.
MER.-ALT. .	Meridian-Altitude.
G.M.T. . .	Greenwich Mean Time.
G.A.T. . .	Greenwich Apparent Time.
S.A.T. . .	Ship Apparent Time.
M.Z.D. . .	Meridian Zenith Distance.
R.A. . . .	Right Ascension.
R.A.M.S. . .	Right Ascension Mean Sun.
G. Sid. T. .	Greenwich Sidereal Time.
S. Sid. T. .	Ship Sidereal Time.
H.A. . . .	Hour Angle.
D.R. LAT. .	Dead Reckoning Latitude.
D.R. LNG. .	Dead Reckoning Longitude.
H.	Difference of Hour Angle and 12h. or 24h.

NAUTICAL USES OF THE SLIDE-RULE.

Hall's Nautical Slide-Rule consists of two slides fitting in grooves on the Rule itself. There are eight scales in all, two on each slide and one on each edge of each groove. These scales are all divided *decimally*, showing sometimes all the tenths of the quantities figured; sometimes only ·2, ·4, ·6, ·8; sometimes merely ·5 or ½. The sub-divisions are carried just as far as they can be conveniently read by the eye. Inspection of the Rule itself will show at once the nature of the graduations. At the top of each scale is printed the name of the quantity it repre[illegible] the value of the graduations.

THE EX-MERIDIAN.

Scales I., II., III., IV., on the left are for use in the Ex-Meridian.

It is well known that the Ex-Meridian is "reduced to the Meridian" by adding to the Altitude a correction depending on LAT., DEC., and Hour Angle, according to the formula

$$X = C \cdot H^2$$

Two steps are necessary, (1) to determine the constant C, depending on LAT. and DEC. only, and being a constant for all Ex-Meridians taken about the same time and of the same body; (2) to multiply H^2 by C.

(1) Scales I. and II. give the value of C, as follows :—

"Set the Zero Mark ∞ (Scale II.) on to the *greater* of LAT. and DEC. Read the factor F which is opposite the other of LAT. and DEC., taken on the same or different side of the Equator according as it is of the same or different name."

It is only for convenience that the greater quantity is read on the upper part of the scale, which is longer than the lower part. It will be found quite easy to allow for fractions of degrees in LAT. and DEC., and to read the factor F correctly to three figures. The range of LAT. and DEC. is from 72° to 55°, N. or S.

EXAMPLE.—To take out F for LAT. 52° 30′ S., DEC. 21° 5′ S.

Set mark ∞ (Scale II.) on to 52½° (Scale I., upper).

Opposite 21·1° (Scale I., same side of Equator), read F = 4·4 just *under*, say 4·38 (Scale II.).

RESULT.—4·4—

EXAMPLE.—To take out F for LAT. 39° 20′ S., DEC. 12° 30′ N.

Set mark ∞ (Scale II.) on to 39⅓° (Scale I., upper).

Opposite 12½° (Scale I., opposite side of Equator), read F = 3·85 exactly (Scale II.).

RESULT—3·85.

(2) Scales III. and IV. give the value of $C \cdot H^2$, as follows : —

"Set the gauge point F (Scale III.) on to the value of F just found (Scale IV.). Then opposite any Hour Angle in minutes and tenths (Scale III.), is found the corresponding value of X in miles and tenths (Scale IV.)."

When the Slide is thus set, it serves for any number of Ex-Meridians of the Sun taken on the same day. The range of Hour Angle is 70 minutes before or after noon. The range of the Correction X is 70′, always (+) to Altitude.

When the nearest ⅓′ in the value of X cannot be read distinctly and with ease, the instrument is automatically

giving a warning that the Hour Angle used is too large for Ex-Meridian treatment, and that consequently the observation is not reliable.

In general, the nearest tenth of a mile can be read and is dependable.

EXAMPLE.—The value of F being 3·65, find the X-Correction for Hour Angles of 10m., 17½m., 25m.

Set F (Scale III.) on to 3·65 (Scale IV.) as in the Illustration.

Opposite 10m. (Scale III.), read 3′ (Scale IV.).

" 17½m. " " 9·2′ "

" 25m. " " 18·7′ "

At about 35m. we should be warned of decreased reliability.

WORKED EXAMPLE. SUN EX-MERIDIAN.

In 52° 30′ N., 4° 45′ W. = 19m., 0s., corrected ALT. of Sun was 58° 12·7′. DEC. 21° 4·7′ N.; EQN. 5m., 56s. – to M.T. G.M.T. by Chron. 23h., 52m., 43s.

A. **Find S.A.T. and H.**

G.M.T.	23h.	52m.	43s.
EQN.	—	5	56
G.A.T.	23	46	47
LNG., W.		19	00
S.A.T.	23	27	47, ∴ H = 32m., 13s.

B. **Find the X-Correction.**

(1) Set mark ∞ (Scale II.) to 52½° (Scale I., upper).

Opposite 21·1° (Scale I., same side of Equator) read F = 4·4, just *under* (Scale II.).

(2) Set gauge point F (Scale III.) to 4·4 (Scale IV.).

Opposite 32·2m. (Scale III.), read X = 37′ (Scale IV.),

∴ X = 37′

C. **Find the Latitude**

Corrected Alt.	58°	12·7′
X-Correction	+	37·0′
	58	49·7
M.Z.D.	31	10·3
Dec	21	04·7
Lat.	52	15·0

RESULT.—Position Point is 52° 15′ N., 4° 45′ W., and the Position Line is drawn through it at right angles to the Sun's Bearing.

WORKED EXAMPLE. STAR EX-MERIDIAN.

In 39° 20′ N., 6° 10′ E. = 24m., 40s., corrected Alt. of Regulus was 63° 02·2′. Dec. 12° 27·4′ N.; R.-A., 10h., 3m., 3s. G.M.T. by Chron., 6h., 37m., 6s.; corrected R.-A.M.S, 2h., 48m., 45s.

A. **Find Star's Hour Angle and H.**

G.M.T.	6h.	37m.	06s.
R.-A.M.S	2	48	45
G. Sid. T.	9	25	51
Lng. E.		24	40
S. Sid. T.	9	50	31
* R.-A.	10	03	03
* H.A.	23	47	28, ∴ H = 12m., 32s.

B. **Find the X-Correction.**

(1) Set mark ∞ (Scale II.) to 39⅓° (Scale I., upper).
Opposite 12½° (Scale I., same side of Equator), read F = 6·7.

(2) Set gauge point F (Scale III.) to 6·7 (Scale IV.).
Opposite 12·5m. (Scale III.), read X = 8·6′ (Scale IV.).
∴ X = 8·6′

C. **Find the Latitude.**

Corrected Alt.	63°	02·2′
X-Correction		+ 8·6
	63	10·8
M.-Z.-D.	26	49·2
Dec.	12	27·4
Lat.	39	16·6

RESULT.—Position Point is 39° 16½′ N., 6° 10′ E., and the Position Line is drawn through it at right angles to the Star's Bearing.

WORKED EXAMPLE: STAR EX-MERIDIAN BELOW POLE.

In this useful case of the Ex-Meridian, the navigator will remember that—

Lat. = Mer.-Alt. + Polar Distance of Star.

In working with the Slide-Rule, the differences from the ordinary routine are :—

(1) Reverse the name of Dec.
(2) H = difference of Star's Hour Angle and 12h.
(3) X-Correction is *minus* to Alt.

In 43° N., 18° W. = 1h., 12m., corrected Alt. below Pole of α *Cassiopeiae* was 9° 8·5′. Dec. 55° 59·7′ N., R.-A. 0h., 34m., 53s. G.M.T. by Chron., 6h., 50m., 56s., corrected R.-A.M.S., 6h., 40m., 27s.

A. **Find Star's Hour Angle, and H.**

G.M.T.	6h.	50m.	56s.
R.-A.M.S.	6	40	27
G. Sid. T.	13	31	23
Lng. W.	1	12	00
S. Sid. T.	12	19	23
* R.-A.		34	53
* H.A.	11	44	30, ∴ H = 15m., 30s.

B. **Find the X-Correction.**

(Lat. 43° N., *reversed* Dec. 56° S.).

(1) Set mark ∞ (Scale II.) to 56° (Scale I.).
Opposite 43° (Scale I., different name), read F = 1·66

(2) Set gauge point F (Scale III.) to 1·66 (Scale IV.).
Opposite 15½m. (Scale III.), read X = 3·3′.

C. **Find the Latitude.**

Corrected Alt.	9°	08·5′
X-Correction	—	3·3
	9	05·2
Polar Distance	34	00·3
Lat.	43	05·5

RESULT.—Position Point is 43° 5½′ N., 18° W., and the Position Line is drawn through it at right angles to the Star's Bearing.

CORRECTION OF ERRORS: POSITION LINES.

The graphic method of plotting Position Lines is often inconvenient, and it is easier to use the alternative method of correcting sights for small errors in assumed data. Thus, if at Ex-Meridian the D.R. Lat. is found to be in error, the Chron. Sight previously worked with the erroneous Lat. has to be corrected. Or again, if the D.R. Lng. used to work an Ex-Meridian be wrong, the resulting Lat. is also wrong and needs correction.

Scales V., VI., VII., VIII. provide these corrections as follows :—

"Set the Lat. (Scale VI.) to the Body's Azimuth (Scale V.). Then corresponding errors in Lat. and Lng. are opposite one another on Scale VII. and VIII."

Since the Position Line is at right angles to the Bearing we name the corrections by the following rule (Mr. Johnson's) :—

"Write down the Bearing, and under it its reverse; then the names of corresponding corrections lie diagonally across the figure."

EXAMPLE.—Morning Chron. Sights in Lat. 47° 11′ N., on a Bearing S. 63° E., worked up to Noon, gave 46° 41·5′ N., 36° 57·7′ W. At Noon, Lat. was found to be 46° 50·0′ N. Find the correct Lng. at Noon.

A. **Find δ LNG.**

δ Lat. = 46° 50′—46° 41½′ = 8·5′ N.
Set Az. 63° (Scale V.) on Lat. 47·2° (Scale VI.).
Opposite 8·5′ (Scale VII.), read δ Lng. = 6·4′ (Scale VIII.).

B. **Name δ LNG.**

Write down the bearing - - -	S.	E.
Under it place its reverse - - -	N.	W.

δ Lat. is N., and diagonally we find N. and E.
∴ name of δ Lng. is E.

RESULT.—Corrected Lng. = 36° 57·7′ W.—6·4′ E. = 36° 51·3′ W.

The Chron. Sight and the Ex-Meridian may be used to correct one another mutually, as follows :—

		Lat.	Lng.	Az.
7.15 a.m.	Chron. Sights	37° 50·0′ N.	69° 21·0′ E.	S. 68° E.
	Run	52·8 S.	20·6 W.	N. W.
11.40 a.m.	Assumed position	36 57·2 N.	69 00·4 E.	
11.40 a.m.	By Ex Meridian	37 05·4 N.	69 00·4 E.	S. 11° E.
	δ Lat. =	8·2′ N.		N. W.

A. **Correct Chron. Sights for error in LAT. = 8·2′ N.**

Set Az. 68° (Scale V.) to Lat. 38° (Scale VI.).
Opposite δ Lat. 8·2′ (Scale VII.), read δ Lng. = 4·2′
Name δ Lng. E., from the N.E. diagonal of the figure.
RESULT.—Lng. = 69° 00·4′ E. + 4·2′ E. = 69° 04·6′ E.

B. **Correct Ex-Mer. for error in LNG.** just found = 4·2′ E.

Set Az. 11° (Scale V.) to Lat. 37° (Scale VI.).
Opposite δ Lng. 4·2′ (Scale VIII.), read δ Lat. = 0·6′
Name δ Lat. N., from the N.E. diagonal of the figure.
RESULT.– Lat. = 37° 05·4′ N. + 0·6′ N. = 37° 06′ N.

If neither of the sights be an Ex-Meridian, we must use the method known as "Johnson's," of which an example is given to show how the Slide-Rule does all the arithmetic automatically. It is supposed that two Star Chronometers have been taken and the results logged as in the form below. The work is shown in heavy type, and the explanation follows.

Obs.	D.R. Lat.	Az.	Factor.	Lng.	δ Lat.	δ Lng.
I.	30°05′ N.	S. 80° W.	**0·2**	40°01·5 W.	N. E.	**1·1′ E.**
II.	30 05 N.	N. 59 E.	**0·7**	40 04·3 W.	S. W.	**3·9′ E.**
			Diff. = 0·5	**δ Lng.=2·8**	**5·6′ S.**	

(A) Take for each Az. and the Lat. the δ Lng due to 1′ of δ Lat.

These are the 0·2 and 0·7 entered as "Factor."

The Azimuths being "*Like*," take the difference of the Factors, namely, 0·5. Take also the δ Lng. = 2·8.

(B) Set the diff. 0·5 (Sc. VII.) on δ Lng. 2·8′ (Sc. VIII.).
Opposite 1·0 „ read δ Lat. = 5·6′ „
„ 1st Factor 0·2 „ „ 1st δ Lng. = 1·1′ „
„ 2nd „ 0·7 „ „ 2nd „ = 3·9′ „

(C) Name the 1st and 2nd δ Lng. by considering that there is only one way of applying them so as to get the same result from each sight. Thus both are E.

(D) Name the δ Lat. by writing above it the diagonal figure for the second sight, using the 2nd Az. and the 2nd δ Lng. Thus δ Lat is S.

RESULT.—Lng. = 40° 01·5′ W. — 1·1′ E. = 40° 00½′ W.,
or = 40 04·3 W. — 3·9 E. = 40° 00½′ W.
Lat. = 30 05·0 N. — 5·6 S. = 29° 59½′ N.

TO FIX DISTANCE BY BEARINGS AND RUN.

Use Scale VI. and Scale VIII. The numbers on VIII. are proportional to the opposite Cosines on VI.

Example.—Given the Course S 85° E, and the run 7·5 miles, 1st bearing (A) of Light (L) N 70° E, and 2nd bearing (B) N 40° E.

In the triangle formed, we have the angles at
A = 25°, L = 30°, A B = 7·5, required L B.
Sin. 25° = Cos. 65°, Sin. 30° = Cos. 60°.
Set 60° (VI.) on to 7·5 (VIII.).
Opposite 65° (VI.) read 6·3 (VIII.). *Result*—L B = 6·3 miles.

As the Scales VI. and VIII. are not contiguous, a little difficulty may be experienced in setting by eye alone. The straight edge of a card, held parallel to the lines on the Scale, will, however, ensure accuracy of setting.

SPECIAL USES OF THE SLIDE-RULE.

DIP.

Scales III. and IV. can be used as a table of DIP.

Set 4·4′ (Scale III.) to 20 feet (Scale IV.).

Then opposite any number of feet on Scale IV., may be read the corresponding DIP on Scale III.

Thus at 25 ft., read DIP. = 4·9′
" 30 " " " = 5·4′
" 36 " " " = 5·9′, etc.

REFRACTION.

Scales V. and VII. can be used to find Refraction.

Set Zero (Scale VI.) on to ALT. (Scale V.).

Then opposite I (Scale VII.), read REF. in miles (Scale VIII.).

Thus for ALT. 40°, read REF. = 1·2′
" " 30° " " = 1·7′
" " 20° " " = 2·7′, etc.

CONVERSION OF DEP. and δ LNG.

Set LAT. (Scale VI.) to the middle * (Scale V.).

Then DEP. (Scale VII.) is opposite δ LNG. (Scale VIII.).

EXAMPLE.—In LAT. 30½°, convert 17·2′ DEP. to δ LNG.

Set 30½° (Scale VI.) to middle * (Scale V.).

Opposite 17·2′ (Scale VII.) read δ LNG. = 20′ (Scale VIII.).

EXAMPLE.—In LAT. 45°, convert 21½′ δ LNG. to DEP.

Set 45° (Scale VI.) to middle * (Scale V.)

Opposite 21½′ (Scale VIII.) read DEP = 15·2′ (Scale VII.).

CORRECTION OF DEC. AND EQN. FOR G.M.T.

We state *rules* only, here; for the *reasons* see the explanation which follows:—

(1). To correct DEC. for the Varn. given in seconds per hour (″/h.) in the Almanac.

"Set 60 (Scale VII.) to the Varn. ″/h. (Scale VIII.). Opposite G.M.T. hours and tenths (Scale VII.), read change, miles and tenths (Scale VIII.)."

EXAMPLE.—Var. = 52·8 ″/h. G.M.T. 4h., 25m.

Set 60 (Scale VII.) to 52·8 (Scale VIII.).

Opposite 4·4h. (Scale VII.), read change = 3·9′ (Scale VIII.)

The setting of 60 to the Varn. reduces seconds to miles, and gives the answer in miles and tenths.

(2). To correct EQN. for the Varn. given in seconds per hour (*s/h*) in the Almanac.

"Set 1 (Scale VII.) to the Varn. *s/h* (Scale VIII.). Opposite G.M.T., hours and tenths (Scale VII.), read change, seconds and tenths (Scale VIII.)."

EXAMPLE.—Varn. = 0·67 *s/h*, G.M.T. = 7h. 37m.

Set 1 (Scale VII.) to 0·67 (Scale VIII.).

Opposite 7·6h. (Scale VII.), read change = 5·1s. (Scale VIII.).

The above explanations and examples cover the most important technical uses of the Slide-Rule in Navigation. We proceed to a more general description of the nature of the instrument and its universal employment as a calculating machine.

THE PRINCIPLE OF THE SLIDE-RULE.

The Slide-Rule does mechanically what Logarithms do arithmetically. The reader is supposed to understand such formulæ as:

$$\text{Log A} + \text{Log B} = \text{Log A B}$$
$$\text{Log A} - \text{Log B} = \text{Log A/B}$$
$$n \text{ Log A} = \text{Log A}^n$$
$$\text{Log A} \div n = \text{Log} \sqrt[n]{A}$$

To take the simplest case possible, let us multiply 2×3.

By logs, we could say

$$\log 2 = 0\cdot30103$$
$$\log 3 = 0\cdot47712$$
$$\log 6 = 0\cdot77815$$

Or marking off 0·301 inches on a Scale, and 0·477 on another, we could put them end to end and measure the combined length = 0·778, and find that this was log 6.

The Slide-Rule is constructed of such Scales.

Consider the Scales VII. and VIII., which are identical.

Starting at the point 1, the distance 1 to 10 is taken as unity, since log 10 = 1. The distances 1 to 2, 1 to 3, 1 to 4, etc., are simply the logs of 2, 3, 4, etc., on the same Scale. The distance 1 to 20 is 1 + log 2, the distance 1 to ·3 is − 1 + log 3, and so on.

To recur to our problem, 2×3.

Set 1 (Scale VII.) to 2 (Scale VIII.).
Opposite 3 (Scale VII.), read 6 (Scale VIII.).

What we have done is to add log 2 + log 3, mechanically, and the result is log 6. Similarly, every number on Scale VIII. is twice its opposite number on Scale VII.

Take now the question $2\cdot1 \times 3\cdot2$.

Set 1 (Scale VII.) to 2·1 (Scale VIII.).
Opposite 3·2 (Scale VII.), read 6·7 + (Scale VIII.).

Common-sense tells us that the something over must be a 2, so the result is $2\cdot1 \times 3\cdot2 = 6\cdot72$.

Going a step further, take $2\cdot16 \times 3\cdot25$.

Here we have to interpolate by eye for the 3rd figure.

Set 1 (Scale VII.) to 2·16 (Scale VIII.).
Opposite 3·25 (Scale VII.), read 7·0 (Scale VIII.).

Here we cannot be positive that the result is not 6·99 or 7·01, but for all practical purposes we know that it is very nearly 7·00. As a matter of fact it is 7·02 accurately, so we are in error by 1 in 350. The Slide-Rule does not pretend to be more accurate than within 1 %, which is good enough for all ordinary practical work.

Take now $7\cdot35 \times 9\cdot05$

Set 1 (Scale VII) to 7·35 (Scale VIII.).
On 9·05 „ read 66·5 „

RESULT.—66·5.

But in the case $8\cdot75 \times 9\cdot4$

Set 1 (Scale VII.) to 8·75 (Scale VIII.).
On 9·4 „ we have overrun the marks.

However, all the parts of the Scales whether marked as ·1, ·2, etc., or 1, 2, etc., or 10, 20, etc., are exactly alike, so going back to ·94 (Scale VII.), we find 8·2 + (Scale VIII.), and we read it as 82 +, shifting the decimal place by common sense.

RESULT.—82 +.

We conclude, then, that we may read our result where we please, and that we may use ·1, 1, or 10 interchangeably in setting, so long as we finally fix our decimal point right in the result.

RULE FOR MULTIPLICATION.

"Set 1 of the *slide* to one of the factors on the *rule*. Opposite the other factor on the *slide* read the product on the *Rule*."

RULE FOR FIXING THE POINT.

"If the first factor and the product are read in the same part of the Scale, the number of digits in the product = the sum of the number of digits in the factors, *minus* 1."

"If to read the product we have to pass from one part of the Scale to another, the number of digits in the product = the sum of the number of digits in the factors."

Here by "parts" of the Scale, we mean the parts from ·1 to 1, 1 to 10, etc. By digits we mean the figures before the decimal point.

EXAMPLE.—32·5 × 29·4.
Set 1 (slide) to 3·25 (rule).
On 2·94 (slide), read 96—(rule).

The product is in the *same* part, ∴ there are 2 + 2—1 digits and result = 960—.

The actual result is 955·5, so we are correct to about ½ %.

RESULT.—960—.

EXAMPLE.—3·65 × 43·2.
Set 1 (slide) to 3·65 (rule).
On 4·32 (slide), read 1575 + (rule).

We have passed out of the part of the Scale where we started, so the number of digits is 1 + 2 = 3, and the result is 157·5 +.

The actual result is 157·7; we are correct to 1 in 1000.

RESULT.—157·5.

The decimal point in the case of multiplication of decimals—and indeed in every other case—can be seen by common sense. It may be well, however, to state a rule not found in arithmetics, but of universal application, and used by all practical calculators.

A number such as 123·456 is written 1·23456, (+ 2), where (+ 2) means that we have 2 tens to multiply by. Similarly ·0987 is written 9·87 (— 2), where (— 2) means that we have 2 tens to divide by.

EXAMPLE.—92,800,000 miles = 9·28 (+ 7) = Distance of Sun.
0·0648 grammes = 6·48 (— 2) = 1 grain.

Consider then 92800 × 0·0648 = 9·28 (+ 4) × 6·48 (— 2).

Result by Slide-rule = 60·1 (+ 4 — 2) = 6010.

The actual result is 6013, with error less than 1 per 1000.

Or again 0·7962 × 0·00327 = 7·962 (— 1) × 3·27 (— 3).

Result by Slide Rule = 26 (— 1 — 3) = 0·0026.

The actual figures are 2603, about 1 per 1000 error.

RULE FOR DIVISION.

"Set the divisor on the *slide* on the dividend on the *rule*. Opposite 1 on the *slide* read the quotient on the *rule*."

This is merely reversing the rule for multiplication.

EXAMPLE.—62 ÷ 4.

Set 4 (slide) on 62 (rule).
On 1 (slide), read 15·5 (rule).

The Slide-Rule itself fixes the place of the point.

RESULT.—15·5.

EXAMPLE.—9·75 ÷ 34·2.

Set 34·2 (slide) on 9·75 (rule).
On 1 (slide), read 0·285 (rule).

The Slide-Rule itself fixes the point.

RESULT.—0·285.

But in cases where the numbers cannot be read directly from the Rule with the point fixed, we state a rule which is that for multiplication reversed.

RULE FOR FIXING THE POINT.

"If the dividend and the quotient are read in the same part of the scale, the number of digits in the quotient = number in dividend — number in divisor, *plus* 1."

"If to read the quotient we have to pass from one part of the Scale to another, the number of digits in the quotient = number in dividend — number in divisor."

"A *minus* number of digits means 'noughts' before the first decimal figure."

EXAMPLE.—12·34 ÷ 567·8.

Set 5·678 (slide) on 1·234 (rule).
On 1 (slide), read 217.

We have passed out of the part of the Scale where we started, so the number of digits is 2 — 3 = — 1, and the result is 0·0217.

EXAMPLE.—0·0139 ÷ 0·00375.

Set 3·75 (slide) on 1·39 (rule).
On 1 (slide) read 371.

We have passed out of our part of the Scale, so the digits are — 1 — (— 2) = + 1, and the result is 3·71.

EXAMPLE.—365·24 ÷ 0·1789.

Set 1·789 (slide) on 3·6524 (rule).
On 1 (slide) read 204.

We are in the same part of the Scale, so the number of digits is 3 — 0 + 1 = 4, and the result is 2040.

In all cases, however, a rough mental estimate will give the correct position by common sense.

PROPORTION.

In any setting of the slide and rule, opposite numbers are proportionals.

Set 4 (slide) on to 3 (rule).
Read 8 opposite 6, 16 opposite 12, 20 opposite 15, etc.

Hence if any proportion question be set out as an equation of two fractions, the missing term can be filled in.

EXAMPLE.—Solve x : 16 :: 24 : 64.

or $\frac{x}{16} = \frac{24}{64}$ numerators on rule. / denominators on slide.

Set 64 (slide) on 24 (rule).
Read x = 6 (rule) on 16 (slide).

EXAMPLE.—Solve 2·75 : x :: 76 : 24.

or $\frac{2{\cdot}75}{x} = \frac{76}{24}$ ... rule. / ... slide.

Set 24 (slide) on 76 (rule).
Read x = ·87 (slide) on 2·75 (rule).

This proportional property enables us to convert measures of length, weight, money, etc., at sight.

EXAMPLE.—If 4·41 lbs. = 2 kilos, reduce 200 lbs. to kilos.

Set 4·41 lbs. (slide) on 2 kilos. (rule).

On 200 lbs. (slide) read 91 kilos. (rule).

RESULT.—91 kilos.

EXAMPLE.—If £1 = 25 fr., 40 cents., find value of 750 francs

Set £1 (slide) on 25·4 fr. (rule).

Read £29·5 (slide) on 750 fr. (rule).

RESULT.—£29 10s.

EXAMPLE.—In 11 hrs. 25 m., got in 982 tons of coal, find the average per hour.

Set 11·4 h. (slide) on 982 (rule).

On 1 h. (slide) read 86 (rule).

RESULT.—86 tons.

EXAMPLE.—A sea mile = 6080 ft. Reduce 12 sea miles to statute miles.

(6080 statute miles = 5280 sea miles).

Set 6080 (slide) on 5280 (rule).

Read 13·8 (slide) on 12 (rule).

RESULT.—13·8 statute.

PERCENTAGES.

Percentages are merely proportions where one term happens to be 100.

EXAMPLE.—$3\frac{1}{2}$ % on £202 15s.

Set $3\frac{1}{2}$ (slide) on 100 (rule).

Read 7·1 (slide) on 202·75 (rule).

RESULT.—£7 2s.

EXAMPLE.—Income Tax on £187 13s., @ 1s. 2d. in the £.

Set 14d. (slide) on 240d. (rule).

Read 10·95 £ (slide) on £187·65 (rule).

RESULT.—£10 19s.

EXAMPLE.—A patent Log overlogs $4\frac{3}{4}$ %. What is the real run for 162′ logged ?

Set 100 (slide) on $104\frac{3}{4}$ (rule).

Read 155 (slide) on 162 (rule).

RESULT.—155′.

COMPLEX FORMULÆ.

Results generally appear in the shape—

$$x = \frac{a.\ b.\ c.\ d.}{p.\ q.\ r.}$$

We take such cases piecemeal. For Example :

$$x = \frac{3 \times 4 \times 5 \times 6}{7 \times 11 \times 13}$$

1st. $\frac{3 \times 4}{7} = a$, or $\frac{3}{7} = \frac{a}{4}$, $a = 1{\cdot}71$

2nd. $\frac{1{\cdot}71 \times 5}{11} = b$, or $\frac{1{\cdot}71}{11} = \frac{b}{5}$, $b = {\cdot}78$

3rd. $x = \frac{{\cdot}78 \times 6}{13}$, or $\frac{{\cdot}78}{13} = \frac{x}{6}$, $x = {\cdot}36$

RESULT.—0·36.

INVERSES OR RECIPROCALS.

The quantity $\frac{1}{x}$ is called the Inverse or Reciprocal of x, and is found by dividing x into unity by ordinary division on the Slide-Rule.

EXAMPLE.— $x = 73{\cdot}2 \left[\frac{1}{1{\cdot}728} - \frac{1}{3{\cdot}125}\right]$

$\frac{1}{1{\cdot}728} = {\cdot}58$ and $\frac{1}{3{\cdot}125} = {\cdot}32$

$\therefore x = 73{\cdot}2[{\cdot}58 - {\cdot}32] = 73{\cdot}2 \times {\cdot}26 = 18{\cdot}9.$

RESULT.—18·9.

EXAMPLE.—$x = \frac{3}{\cdot794} + 2 + \frac{1}{1\cdot26}$

$= 3\cdot78 + 2 + \cdot79 = 6\cdot57.$

RESULT.—6·57.

EXAMPLE.—$\frac{1}{30\cdot6} = x\left[\frac{1}{30\cdot4} - \frac{1}{34\cdot5}\right]$

$\cdot0327 = x\,[\,\cdot0329 - \cdot0290\,]$

$\cdot0327 = x \times \cdot0039$

$x = 8\cdot4$

RESULT.—8·4.

SQUARE ROOTS.

Scales III. and IV. can be used as a table of square roots. For if we set 10 (Scale III.) on 1 (Scale IV.) and read this 1 as being 100, the 2 as 200, etc., we find :

Scale III., square roots.				Scale IV., Numbers.
4	...	...	...	16
5	...	...	...	25
7·74	...	...	...	60
8·35	...	...	...	70
21·2	...	...	...	450
28·2 etc.		...	...	795 etc.

Obviously we can read the squares of numbers by the same setting.

EXAMPLE.—$(1\cdot32)^2 \div \sqrt{7\cdot54}$

$= 1\cdot74 \div 2\cdot75 = \cdot635.$

POWERS AND ROOTS.

Assuming the formulæ :

$\text{Log } A^n = n \,.\, \log A$, and $\log \sqrt[n]{A} = \frac{1}{n} \log A.$

We can use the proportional compasses to take fractional or other roots and powers.

EXAMPLE.—$\sqrt[3]{18}$.

Set the Compasses at 1 : 3.

Take off the distance 1 to 18, reverse the compasses, and measure off one-third of the distance. The result is 2·62.

If the compasses were not at hand, we could measure the actual distance 1 to 18 = 5·01 inches, divide by 3 and get 1·67, corresponding to 2·62.

EXAMPLE.—$(3\cdot54)^{\frac{3}{4}}$

Distance 1 to 3·54 = 2·19

¾ of this = 1·64, corresponding to 2·58.

EXAMPLE.—$(\cdot372)^{\frac{3}{5}}$

Distance back from 1 to ·372 = — 1·72

⅗ of this = — 1·03, corresponding to ·552.

FORMULÆ OF ÆTHERIC TELEGRAPHY.

These formulæ are, perhaps, as troublesome as any that can be constructed, and they yield readily to the Slide-Rule. The conversion of feet into centimetres (1 ft. = 30·4 cm.) is done at sight.

The **Capacity Formula** is $S = l \div [4\cdot6 \log D/d]$.

Let l = 149 ft. = 4530 cm.
D = 35 ft. = 1065 cm.
d = ·27 cm.

1st. $D/d = 1065 \div \cdot27 = 3950$, whose log = 3·60.

2nd. $S = \frac{4530}{4\cdot6 \times 3\cdot6} = 274.$

The **Self-Induction Formula** is

$L = 2\,l\,[2\cdot3 \log 4\,l/d - 1\cdot1].$

Let l = 200 ft. = 6080 cm.,
,, d = ·3 cm.

1st. $4\,l/d = \dfrac{4 \times 6080}{\cdot 3} = 81{,}000$, whose log $= 4\cdot91$.

2nd. $2\cdot3 \times 4\cdot91 - 1\cdot1 = 11\cdot3 - 1\cdot1 = 10\cdot2$.

3rd. $L = 2 \times 6080 \times 10\cdot2 = 124{,}000$.

The **Frequency Formula** is $n = \dfrac{9\cdot6 \times 10^9}{\sqrt{L.S.}}$

Let L = 123,789, and S = 370.

1st. $L\,S = 1\cdot23789\,(+5) \times 3\cdot70\,(+2) = 4\cdot58\,(+7)$
$= 45\cdot8 \times 10^6$
$\sqrt{L\,S} = 6\cdot77 \times 10^3$

2nd. $n = \dfrac{9\cdot6 \times 10^9}{6\cdot77 \times 10^3} = 1\cdot42 \times 10^6 = 1{,}420{,}000$.

TRIGONOMETRICAL FORMULÆ.

Scale V. is a Scale of L. TANGENTS.

Set 1 (Scale VI.) on 45° (Scale V.),
Read 1 (Scale VII.) on 1 (Scale VIII.). Tan. 45° = 1.

Set 1 (Scale VI). on 40° (Scale V.),
Read ·84 (Scale VII.) on 1 (Scale VIII.). Tan. 40° = ·84.

Reversing the process to get COTANGENTS.
On 1 (Scale VII.), read 1·19 (Scale VIII.). Cot. 40° = 1·19.

RULE.—In the formula x = y . tan. A, if zero of Scale VI. be on A of Scale V., then x of Scale VII. is on y of Scale VIII.

EXAMPLE.—14 = y tan. 37½°.
Set zero (Scale VI.) on 37½° (Scale V.)
On 14 (Scale VII.), read 18¼ (Scale VIII.).

RESULT.—18¼.

And so for any other example involving TAN. or COT.

Scale VI. is a Scale of L. SECANTS.

Set 60° (Scale VI.) on the * (Scale V.).
On 1 (Scale VII.), read 2 (Scale VIII.). Sec. 60° = 2.

Set 48° (Scale VI.) on the * (Scale V.).
On 1 (Scale VII.), read 1·49 (Scale VIII.). Sec. 48° = 1·49.

Reversing the process to find COSINES.
Read ·67 (Scale VII.) on 1 (Scale VIII.). Cos. 48° = ·67.

RULE.—In the formula x = y · cos. A, if the * of Scale V. be on A, then x of Scale VII. is on y of Scale VIII.

EXAMPLE.—13 = 29 · cos. A.
Set 13 (Scale VII.) on 29 (Scale VIII.).
On * (Scale V.), read 63⅓° (Scale VI.).

RESULT.—63⅓°

By using the Scales in combination, and remembering the formula—

δ LNG. = δ LAT. · cot. Az. · sec. LAT.,

we can evaluate any formula of the shape

x = y · cot. A · sec. B,

by setting the Scales as in finding δ LNG., etc., in correcting sights for errors in data.

EXAMPLE.—27· sin.40° = x · tan. 52°
Write 27 = x · tan. 52° · cox. 40°
or 27 = x · cot. 38° · sec. 50°

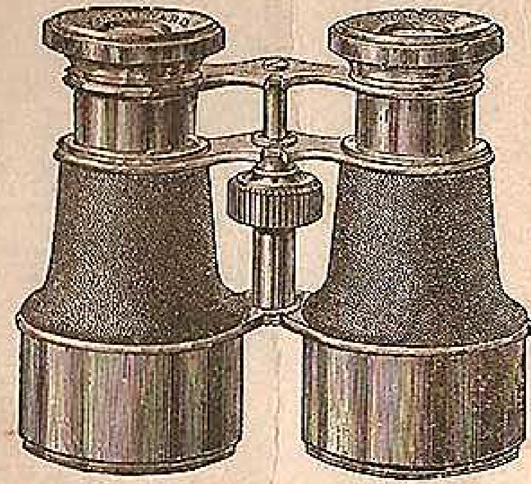
Set 38° AZ. (Scale V.) on 50° LAT. (Scale VI.).
on 27 δ LNG. (Scale VIII.), read 13·5 (Scale VII.).

RESULT.—13·5.

In conclusion, we advise the reader to constantly use the rule, setting himself a simple problem of which he knows the answer, and seeing how to get it from the rule; and then to advance to complicated questions. An hour's practice, puzzling out the reasons of things, is a very real exercise in mathematical common sense.

MARINE BINOCULAR.

THE "CAPTAIN."



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